

# Optimal and Learning Control for Autonomous Robots

## Lecture 13



Jonas Buchli  
Agile & Dexterous Robotics Lab



# Class logistics

Exercise 3 Due: Tue. May 26 - midnight

Interviews: Thu/Fri. May 28/29

<https://ethz.doodle.com/bsi7gvkycvrmht6t>

# LI3

# Review on: Gradient descent methods

# Analytical optimum

Minima (and maxima) of functions

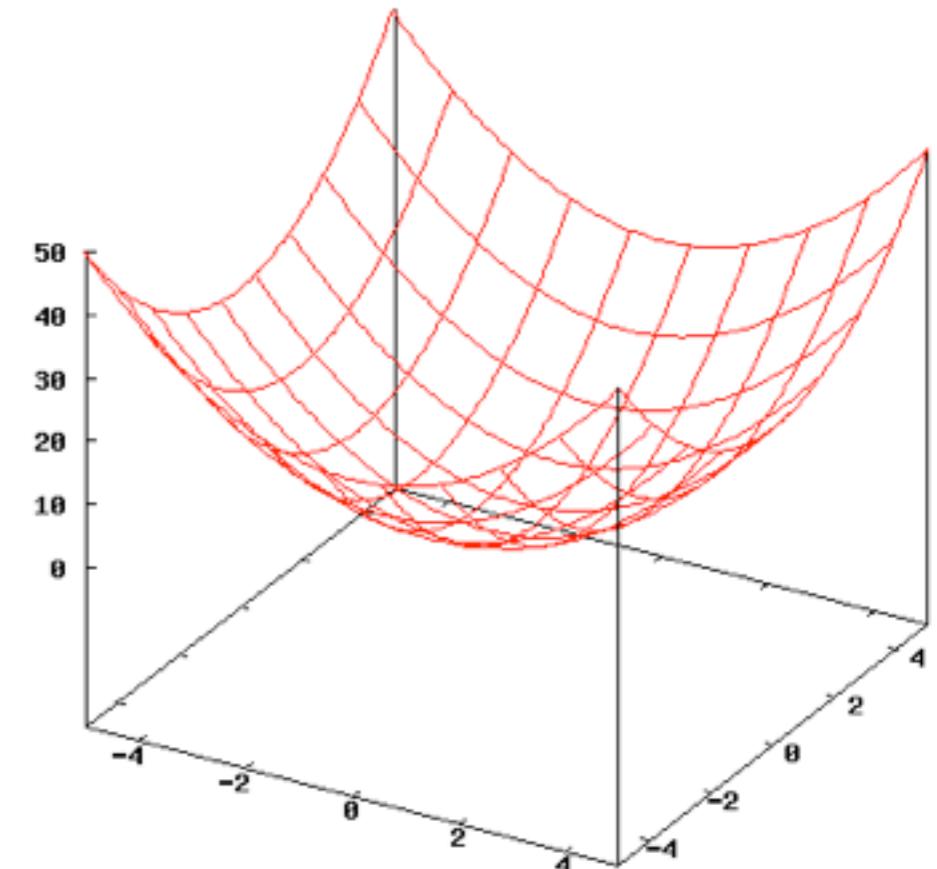
n-dimensional:  $C = f(x_1, \dots, x_n)$

$$\frac{\partial C}{\partial x_i}$$

$$\frac{\partial C}{\partial x_i} = 0$$

$$\nabla C = \left[ \frac{\partial C}{\partial x_i}, \dots, \frac{\partial C}{\partial x_i} \right]^T$$

$$\nabla C = 0$$

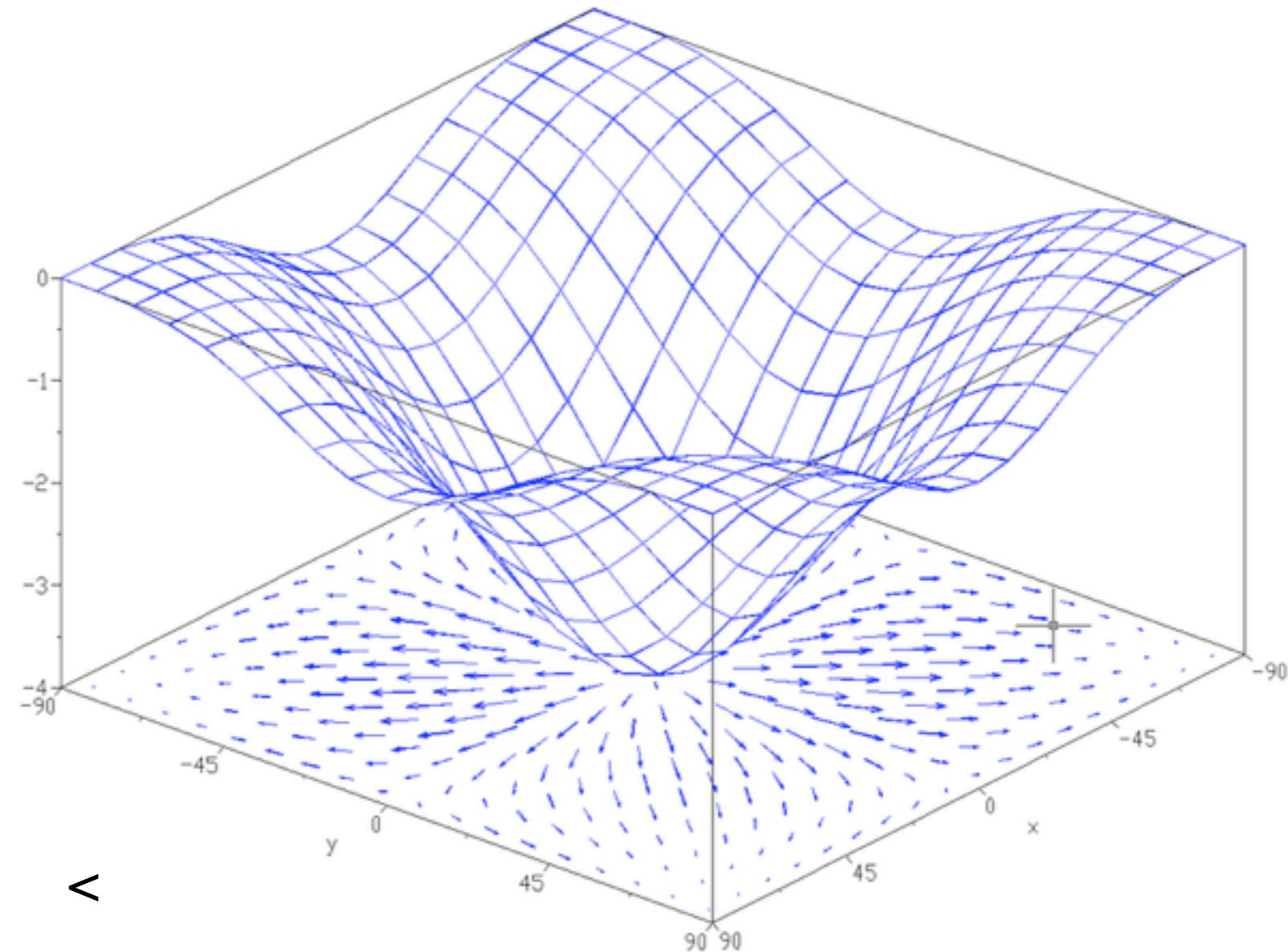


Minimum is an ‘inflection point’  
- slope is 0

Buchli - OLCAR - 2015

# Gradient descent

- Start from initial  $x_0$
  - Loop until convergence
    - $x_{m+1} = x_m - \alpha_m \left[ \frac{\partial C(x)}{\partial x} \right]_{x_m}$
- Learning Rate



**Idea:** Go to the direction where  $C$  decreases, negative direction of gradient vector

# Optimal Control problem with parameterized policy

What if instead of finding the optimal control input (which is a function) we find an approximation of that (function approximation)

$$J_{\text{cost}} = \Phi(t_f) + \int_0^{t_f} L(x_t, u_t) dt$$

$$\text{s.t. } \dot{x} = f(x, u)$$

$$J_{\text{cost}} = \Phi(x_{t_f}) + \sum_{k=t_0}^{t_f} L(x_k, u_k)$$

$$\text{s.t. } x_{n+1} = f(x_n, u_n)$$

$$u(n, x, \theta)$$

$$\theta = [\theta_1, \dots, \theta_p]^T \quad p \ll \frac{t_f}{\Delta t}$$

# Policy Parameterization

Optimal Control problem:

$$J = \phi[\mathbf{x}(t_f), t_f] + \int_{t_0}^{t_f} \mathcal{L}[\mathbf{x}(t), \mathbf{u}(t), t] dt$$

w.r.t

$$\dot{\mathbf{x}}(t) = \mathbf{f}[\mathbf{x}(t), \mathbf{u}(t), t], \quad \mathbf{x}(t_0) = \mathbf{x}_0$$

Parameterize the control

$$\mathbf{u}(t) = \mathbf{u}(\mathbf{k}, t), \quad t_0 \leq t \leq t_f$$

$$\mathbf{u}(\mathbf{k}, t) = \mathbf{k}_1 + \mathbf{k}_2 t + \mathbf{k}_3 t^2 + \dots$$

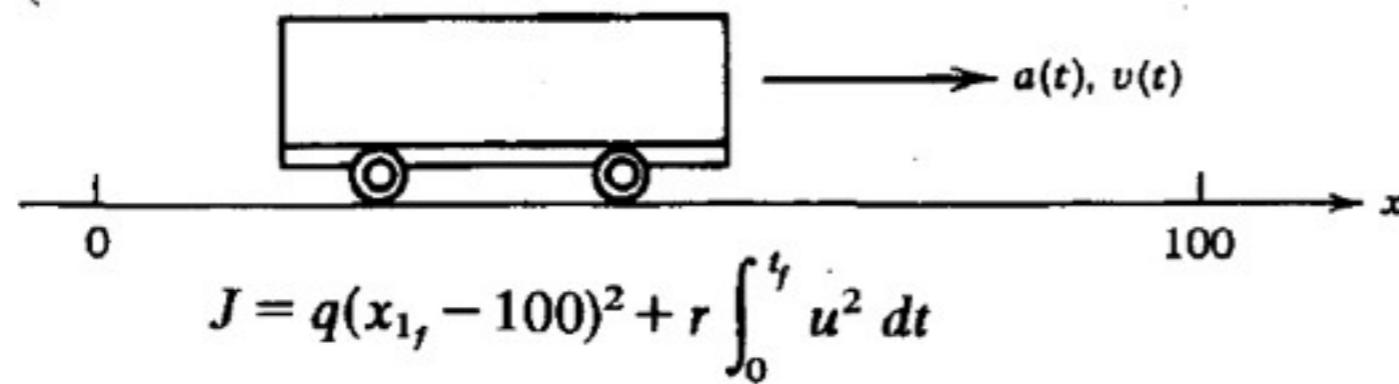
$$\mathbf{u}(\mathbf{k}, t) = \sum_{i=1}^N \left[ \mathbf{k}_{1i} \sin \frac{i\pi t}{(t_f - t_0)} + \mathbf{k}_{2i} \cos \frac{i\pi t}{(t_f - t_0)} \right]$$

If we can write **cost** as  
an **explicit function** of  
**parameter vector**



$$\frac{\partial J}{\partial \mathbf{k}} = \mathbf{0}$$

# Example: Cart on a track



$$\begin{aligned} \dot{x} &= v & \dot{x}_1 &= x_2 \\ \dot{v} &= \frac{f}{m} & \dot{x}_2 &= u \end{aligned}$$

$$\begin{aligned} u &= k_1 + k_2 t & \xrightarrow{\hspace{2cm}} & x_1(t) = \frac{k_1 t^2}{2} + \frac{k_2 t^3}{6} & \xrightarrow{\hspace{2cm}} & J = q[(50k_1 + 166.7k_2) - 100]^2 + \\ & & & x_2(t) = k_1 t + \frac{k_2 t^2}{2} & & r(10k_1^2 + 100k_1k_2 + 333.3k_2^2) \end{aligned}$$

$$\begin{aligned} \frac{\partial J}{\partial k_1} &= k_1(500q + 2r) + k_2(166.7q + 10r) - 1000q & \xrightarrow{\hspace{2cm}} & \begin{bmatrix} (500q + 2r) & (1666.7q + 10r) \\ (1666.7q + 10r) & (5555.6q + 66.7r) \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} 1000 \\ 3333.3 \end{bmatrix} q \\ \frac{\partial J}{\partial k_2} &= k_1(1666.7q + 10r) + k_2(5555.6q + 66.7r) - 3333.3q & & \end{aligned}$$

$\xrightarrow{\hspace{2cm}}$

$$\mathbf{A}\mathbf{k} = \mathbf{B}q$$

$$\mathbf{k} = \mathbf{A}^{-1}\mathbf{B}q$$

# Analytical gradient descent

$$J_{reward} = \sum_{k=0}^H \gamma^k r(x_k, u_k)$$

$$s.t. \quad x_{n+1} = f(x_n, u_n)$$

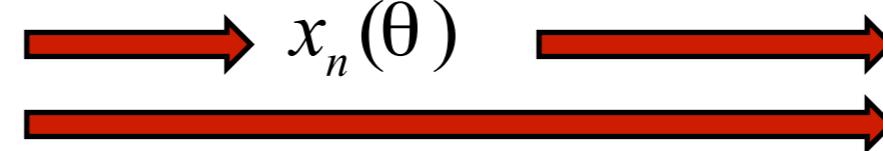
Policy parameterization

$$u_n = u(n, x; \theta)$$

Solve the system dynamics analytically

$$x_{n+1} = f(x_n, u_n)$$

$$u_n = u(n, x; \theta)$$



$$J_{reward}(\theta)$$

# Simple gradient descent

---

**Algorithm 11** Gradient Descent Algorithm
 

---

**given**

A method to compute  $\nabla_{\theta} J(\theta)$  for all  $\theta$

An initial value for the parameter vector:  $\theta \leftarrow \theta_0$

**repeat**

  Compute the cost function gradient at  $\theta$

$$\mathbf{g} = \nabla_{\theta} J(\theta)$$

  Update the parameter vector

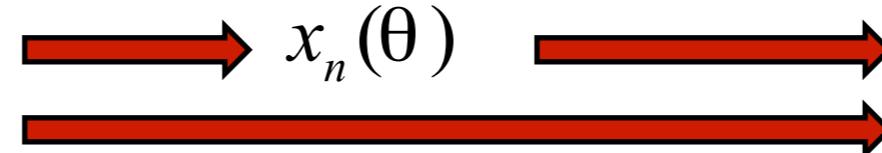
$$\theta \leftarrow \theta - \omega \mathbf{g}$$

**until** convergence
 

---

$$x_{n+1} = f(x_n, u_n)$$

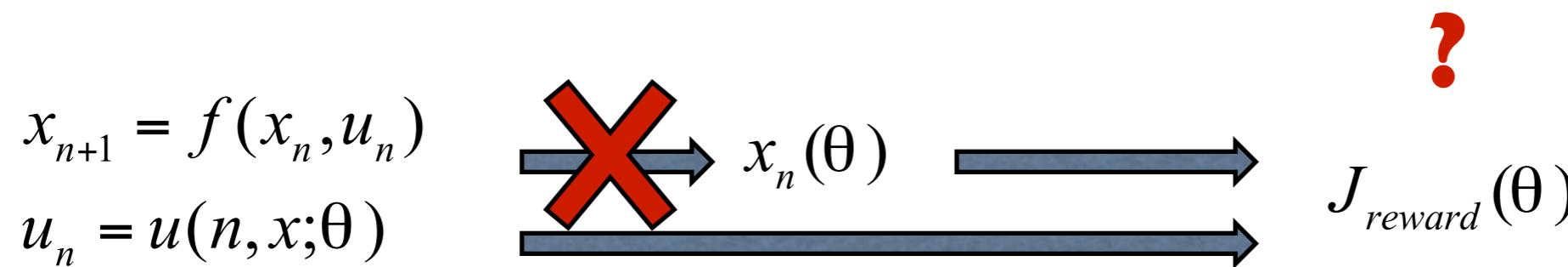
$$u_n = u(n, x; \theta)$$



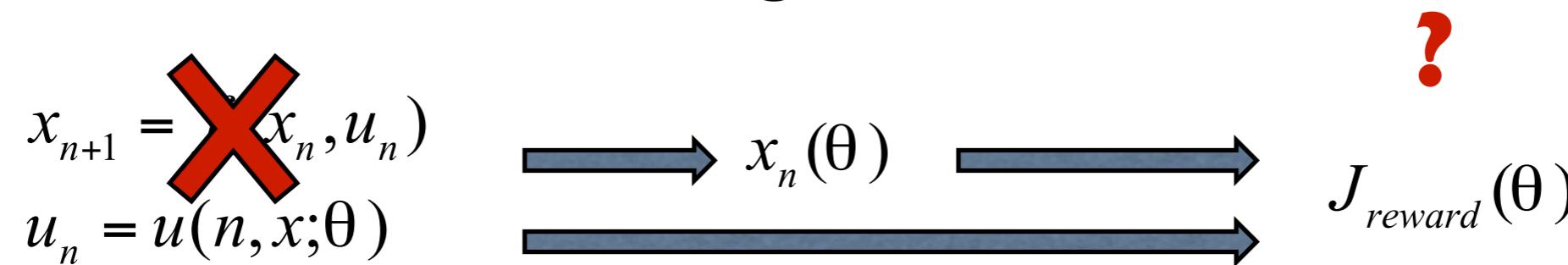
$$J_{reward}(\theta)$$

# Why analytic GD might not be feasible...

- For a general nonlinear system it is not possible



- If the model is not given



# Black box/Model free gradient descent

What if I have no model? i.e.  $C = f(x)$  is not known.

Estimate the gradient       $g = [\nabla J_{reward}(\theta)]_{\theta_m}$

Use the environment as ‘model’: probe, and update based on experience

# Estimation of gradient

$$\frac{dJ(\theta)}{d\theta} = \lim_{d\theta \rightarrow 0} \frac{J(\theta + d\theta) - J(\theta)}{d\theta} \quad \text{OR} \quad \frac{dJ(\theta)}{d\theta} = \lim_{d\theta \rightarrow 0} \frac{J(\theta + d\theta/2) - J(\theta - d\theta/2)}{d\theta}$$

$$\frac{dJ(\theta)}{d\theta} \approx \frac{J(\theta + \Delta\theta) - J(\theta)}{\Delta\theta} \quad \text{one-sided estimate}$$

$$\frac{dJ(\theta)}{d\theta} \approx \frac{J(\theta + \Delta\theta/2) - J(\theta - \Delta\theta/2)}{\Delta\theta} \quad \text{two-sided estimate}$$

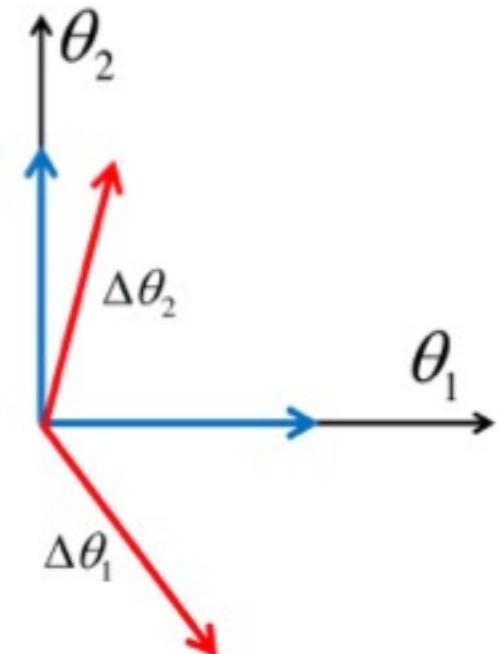
$$\nabla J(\theta_1, \theta_2, \dots, \theta_p) = \left[ \frac{\partial J}{\partial \theta_1}, \frac{\partial J}{\partial \theta_2}, \dots, \frac{\partial J}{\partial \theta_p} \right]$$

**Leads to Finite Difference Method**



# Example

Assume two parameters  $\theta = [\theta_1 \quad \theta_2]^T$



Thus need three samples of the cost:

(e.g. measurement at point of interest, and two one-sided estimates)

$$J(\boldsymbol{\theta}), J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_1), J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_2)$$

$$\frac{dJ(\boldsymbol{\theta})}{d\boldsymbol{\theta}} \approx \frac{J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}) - J(\boldsymbol{\theta})}{\Delta\boldsymbol{\theta}}$$

$$\nabla J(\theta_1, \theta_2, \dots, \theta_p) = \left[ \frac{\partial J}{\partial \theta_1}, \frac{\partial J}{\partial \theta_2}, \dots, \frac{\partial J}{\partial \theta_p} \right]$$

$$J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_1) \approx J(\boldsymbol{\theta}) + \Delta\boldsymbol{\theta}_1^T \nabla J(\boldsymbol{\theta})$$

$$J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_2) \approx J(\boldsymbol{\theta}) + \Delta\boldsymbol{\theta}_2^T \nabla J(\boldsymbol{\theta})$$

$$J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_1) \approx J(\boldsymbol{\theta}) + \Delta\boldsymbol{\theta}_1^T \nabla J(\boldsymbol{\theta})$$

$$J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_2) \approx J(\boldsymbol{\theta}) + \Delta\boldsymbol{\theta}_2^T \nabla J(\boldsymbol{\theta})$$

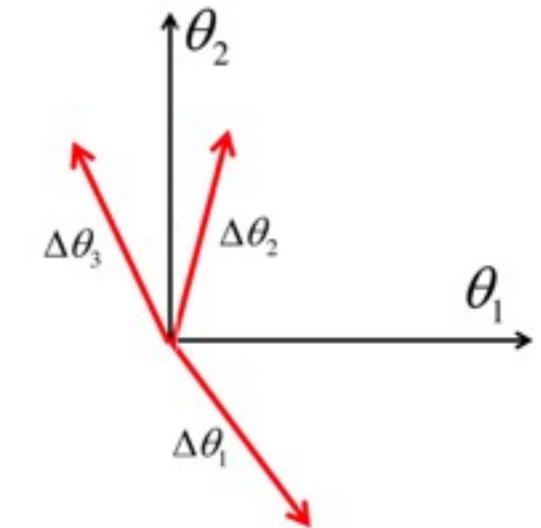
$$\begin{bmatrix} \Delta\boldsymbol{\theta}_1^T \\ \Delta\boldsymbol{\theta}_2^T \end{bmatrix} \nabla J(\boldsymbol{\theta}) = \begin{bmatrix} J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_1) - J(\boldsymbol{\theta}) \\ J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_2) - J(\boldsymbol{\theta}) \end{bmatrix}$$

if perturbations not parallel

$\begin{bmatrix} \Delta\boldsymbol{\theta}_1^T \\ \Delta\boldsymbol{\theta}_2^T \end{bmatrix}$  is a 2-by-2 invertible matrix

$$\nabla J(\boldsymbol{\theta}) = \begin{bmatrix} \Delta\boldsymbol{\theta}_1^T \\ \Delta\boldsymbol{\theta}_2^T \end{bmatrix}^{-1} \begin{bmatrix} J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_1) - J(\boldsymbol{\theta}) \\ J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_2) - J(\boldsymbol{\theta}) \end{bmatrix}$$

# Example 2



Three one sided perturbations

$$J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_1), J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_2), J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_3)$$

$$J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_1) = J(\boldsymbol{\theta}) + \Delta\boldsymbol{\theta}_1^T \nabla J(\boldsymbol{\theta})$$

$$J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_2) = J(\boldsymbol{\theta}) + \Delta\boldsymbol{\theta}_2^T \nabla J(\boldsymbol{\theta})$$

$$J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_3) = J(\boldsymbol{\theta}) + \Delta\boldsymbol{\theta}_3^T \nabla J(\boldsymbol{\theta})$$

unknown  $J(\boldsymbol{\theta})$  and  $\nabla J(\boldsymbol{\theta})$

$$J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_1) = J(\boldsymbol{\theta}) + \Delta\boldsymbol{\theta}_1^T \nabla J(\boldsymbol{\theta})$$

$$J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_2) = J(\boldsymbol{\theta}) + \Delta\boldsymbol{\theta}_2^T \nabla J(\boldsymbol{\theta})$$

$$J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_3) = J(\boldsymbol{\theta}) + \Delta\boldsymbol{\theta}_3^T \nabla J(\boldsymbol{\theta})$$

in matrix form

$$\begin{bmatrix} \Delta\boldsymbol{\theta}_1^T & 1 \\ \Delta\boldsymbol{\theta}_2^T & 1 \\ \Delta\boldsymbol{\theta}_3^T & 1 \end{bmatrix} \begin{bmatrix} \nabla J(\boldsymbol{\theta}) \\ J(\boldsymbol{\theta}) \end{bmatrix}_{\text{unknown}} = \begin{bmatrix} J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_1) \\ J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_2) \\ J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_3) \end{bmatrix}$$

if perturbations pair-wise independent, invertible:

$$\begin{bmatrix} \nabla J(\boldsymbol{\theta}) \\ J(\boldsymbol{\theta}) \end{bmatrix} = \begin{bmatrix} \Delta\boldsymbol{\theta}_1^T & 1 \\ \Delta\boldsymbol{\theta}_2^T & 1 \\ \Delta\boldsymbol{\theta}_3^T & 1 \end{bmatrix}^{-1} \begin{bmatrix} J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_1) \\ J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_2) \\ J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_3) \end{bmatrix}$$

# General FD

General case for p+1 samples:

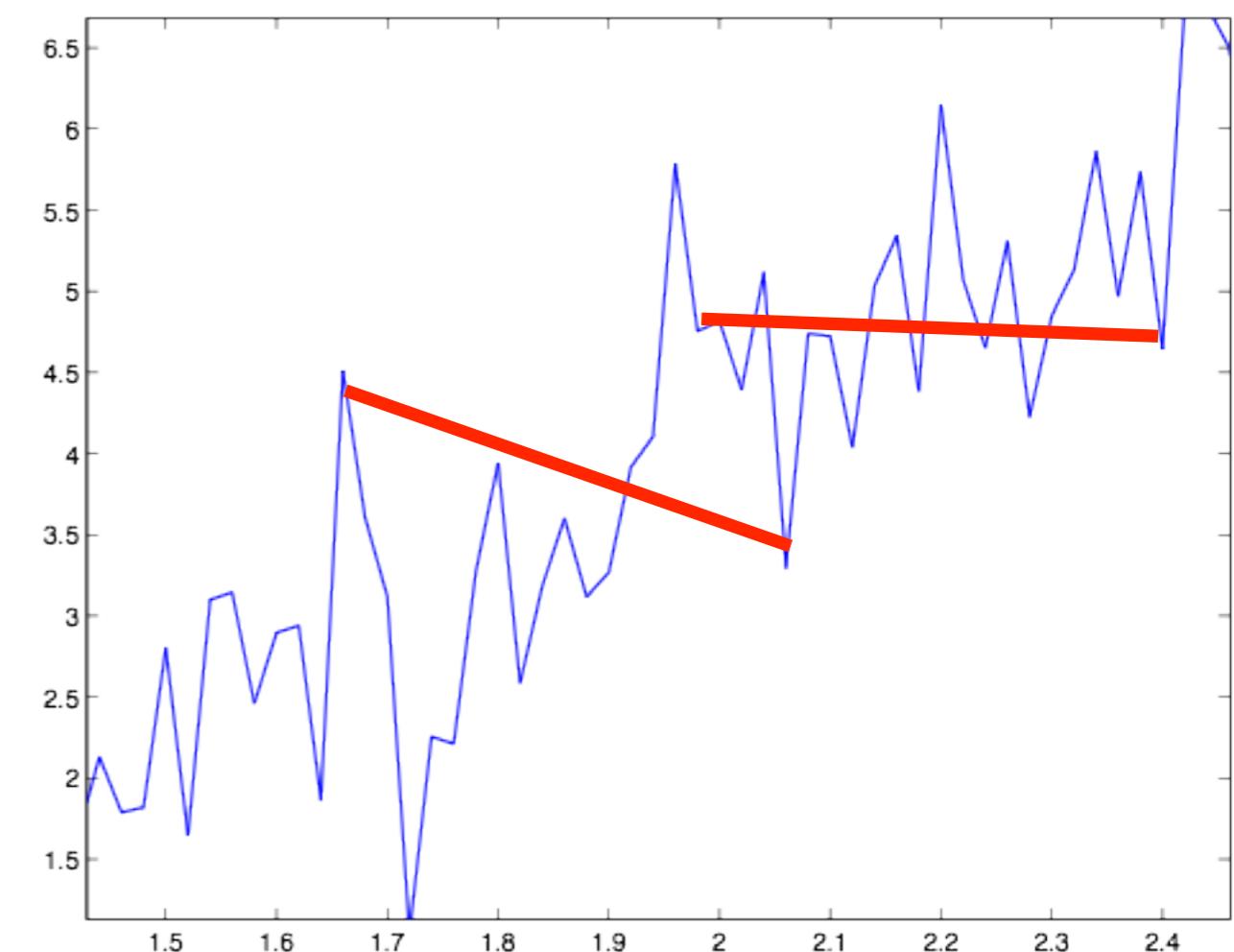
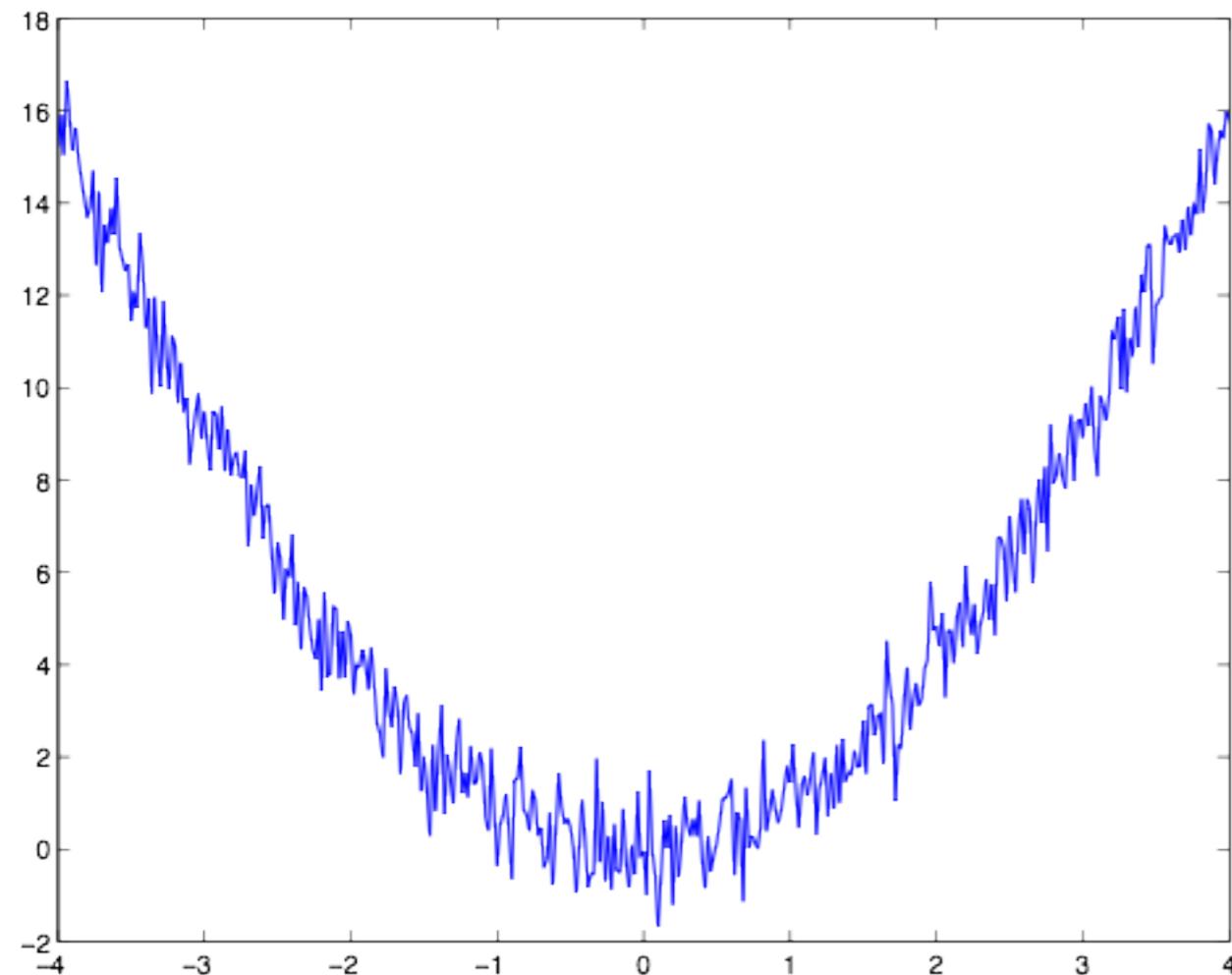
$$\begin{aligned}
 J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_1) &= J(\boldsymbol{\theta}) + \Delta\boldsymbol{\theta}_1^T \nabla J(\boldsymbol{\theta}) \\
 J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_2) &= J(\boldsymbol{\theta}) + \Delta\boldsymbol{\theta}_2^T \nabla J(\boldsymbol{\theta}) \\
 &\vdots \\
 J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_p) &= J(\boldsymbol{\theta}) + \Delta\boldsymbol{\theta}_p^T \nabla J(\boldsymbol{\theta}) \\
 J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_{p+1}) &= J(\boldsymbol{\theta}) + \Delta\boldsymbol{\theta}_{p+1}^T \nabla J(\boldsymbol{\theta})
 \end{aligned}
 \implies \underbrace{\begin{bmatrix} J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_1) \\ J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_2) \\ \vdots \\ J(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_{p+1}) \end{bmatrix}}_{\mathbf{J}} = \underbrace{\begin{bmatrix} \Delta\boldsymbol{\theta}_1^T & 1 \\ \Delta\boldsymbol{\theta}_2^T & 1 \\ \vdots & \\ \Delta\boldsymbol{\theta}_{p+1}^T & 1 \end{bmatrix}}_{\Delta\boldsymbol{\Theta}} \begin{bmatrix} \nabla J(\boldsymbol{\theta}) \\ J(\boldsymbol{\theta}) \end{bmatrix}$$

$$\begin{bmatrix} \nabla J(\boldsymbol{\theta}) \\ J(\boldsymbol{\theta}) \end{bmatrix} = \Delta\boldsymbol{\Theta}^{-1} \mathbf{J}$$

# Stochastic Case

# Stochastic problems

## Noisy cost function



What about gradient now?

Buchli - OLCAR - 2015



# Stochastic FD

Return for a single rollout

$$R = \Phi(\mathbf{x}(N)) + \sum_{k=0}^{N-1} L_k (\mathbf{x}(k), \mathbf{u}(k))$$

Cost is expected return

$$J = E[R]$$

Approximate return by averaging (K rollouts)

need  $K \times (p + 1)$  evaluations of the cost function

# Expected gradient

need  $K \times (p + 1)$  evaluations of the cost function

$$\underbrace{\begin{bmatrix} \frac{1}{K} \sum_{k=1}^K R^k(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_1) \\ \frac{1}{K} \sum_{k=1}^K R^k(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_2) \\ \vdots \\ \frac{1}{K} \sum_{k=1}^K R^k(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_{p+1}) \end{bmatrix}}_{\mathbf{J}} = \underbrace{\begin{bmatrix} \Delta\boldsymbol{\theta}_1^T & 1 \\ \Delta\boldsymbol{\theta}_2^T & 1 \\ \vdots & \\ \Delta\boldsymbol{\theta}_{p+1}^T & 1 \end{bmatrix}}_{\Delta\boldsymbol{\Theta}} \begin{bmatrix} \nabla J(\boldsymbol{\theta}) \\ J(\boldsymbol{\theta}) \end{bmatrix}$$

$$\begin{bmatrix} \nabla J(\boldsymbol{\theta}) \\ J(\boldsymbol{\theta}) \end{bmatrix} = \Delta\boldsymbol{\Theta}^{-1} \mathbf{J}$$

Instead of doing many  
(similar) perturbations

$$N \leq K \times (p + 1)$$

$$\underbrace{\begin{bmatrix} R(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_1) \\ R(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_2) \\ \vdots \\ R(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_N) \end{bmatrix}}_{\mathbf{R}} = \underbrace{\begin{bmatrix} \Delta\boldsymbol{\theta}_1^T & 1 \\ \Delta\boldsymbol{\theta}_2^T & 1 \\ \vdots & \\ \Delta\boldsymbol{\theta}_N^T & 1 \end{bmatrix}}_{\Delta\boldsymbol{\Theta}} \begin{bmatrix} \nabla J(\boldsymbol{\theta}) \\ J(\boldsymbol{\theta}) \end{bmatrix}$$

$\Delta\boldsymbol{\Theta}$  is  $N \times (p + 1)$       If  $N \geq p + 1$

$\Delta\boldsymbol{\Theta}$  rank  $p + 1$

use left pseudoinverse

$$\begin{bmatrix} \nabla J(\boldsymbol{\theta}) \\ J(\boldsymbol{\theta}) \end{bmatrix} = \Delta\boldsymbol{\Theta}^\dagger \mathbf{R} = (\Delta\boldsymbol{\Theta}^T \Delta\boldsymbol{\Theta})^{-1} \Delta\boldsymbol{\Theta}^T \mathbf{R}$$



# Finite difference - general

$$\begin{bmatrix} \nabla J(\boldsymbol{\theta}) \\ J(\boldsymbol{\theta}) \end{bmatrix} = \Delta\boldsymbol{\Theta}^\dagger \mathbf{R} = (\Delta\boldsymbol{\Theta}^T \Delta\boldsymbol{\Theta} + \lambda \mathbf{I})^{-1} \Delta\boldsymbol{\Theta}^T \mathbf{R}$$

$$\mathbf{R} = \begin{bmatrix} R(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_1) \\ R(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_2) \\ \vdots \\ R(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_N) \end{bmatrix}, \quad \Delta\boldsymbol{\Theta} = \begin{bmatrix} \Delta\boldsymbol{\theta}_1^T & 1 \\ \Delta\boldsymbol{\theta}_2^T & 1 \\ \vdots & \\ \Delta\boldsymbol{\theta}_N^T & 1 \end{bmatrix}$$

---

**Algorithm 12** Gradient Descend Algorithm with Finite Difference Method

---

**given**

The cost function:

$$J = E \left[ \Phi(\mathbf{x}(N)) + \sum_{k=0}^{N-1} L_k (\mathbf{x}(k), \mathbf{u}(k)) \right]$$

A policy (function approximation) for the control input:  $\mathbf{u}(n, \mathbf{x}) = \mu(n, \mathbf{x}; \boldsymbol{\theta})$

An initial value for the parameter vector:  $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta}_0$

The parameter exploration standard deviation:  $c$

The regularization coefficient:  $\lambda$

The learning rate:  $\omega$

**repeat**

Create  $N$  rollouts of the system with the perturbed parameters  $\boldsymbol{\theta} + \Delta\boldsymbol{\theta}$ ,  $\Delta\boldsymbol{\theta} \sim \mathcal{N}(\mathbf{0}, c^2\mathbf{I})$

Calculate the return from the initial time and state for the  $n$ th rollout:

$$R(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_n) = \Phi(\mathbf{x}(N)) + \sum_{k=0}^{N-1} L_k (\mathbf{x}(k), \mathbf{u}(k))$$

Construct  $\mathbf{R}$  and  $\boldsymbol{\Theta}$  matrices as:

$$\mathbf{R}_{N \times 1} = [R(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}_n)]_n, \boldsymbol{\Theta}_{N \times (p+1)} = [\Delta\boldsymbol{\theta}_n^T \ 1]_n$$

Calculate the value and gradient of the cost function at  $\boldsymbol{\theta}$

$$\begin{bmatrix} \nabla J(\boldsymbol{\theta}) \\ J(\boldsymbol{\theta}) \end{bmatrix} = (\Delta\boldsymbol{\Theta}^T \Delta\boldsymbol{\Theta} + \lambda \mathbf{I})^{-1} \Delta\boldsymbol{\Theta}^T \mathbf{R}$$

Update the parameter vector:

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \omega \nabla J(\boldsymbol{\theta})$$

**until** convergence

---

# Learning Rate & Stability of gradient descent

## Stability of maps

$$x_{m+1} = x_m - \gamma_m \left[ \frac{\partial C(x)}{\partial x} \right]_{x_m} \Leftrightarrow x_{m+1} = T(x_m)$$

Small perturbation should be damped out

$$x'_m = x_m + \Delta x_m$$

$$x'_{m+1} = x_{m+1} + \Delta x_{m+1}$$

$$\Delta x_{m+1} = \left[ \frac{dT}{dx} \right]_{x_m} \Delta x_m$$

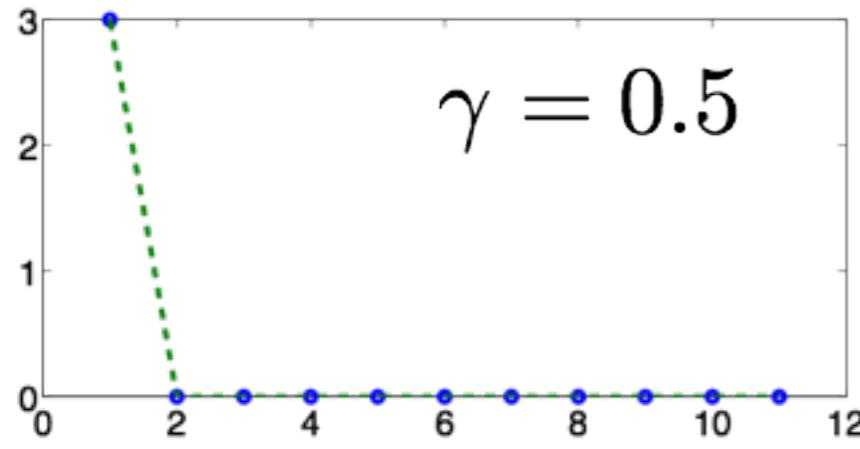
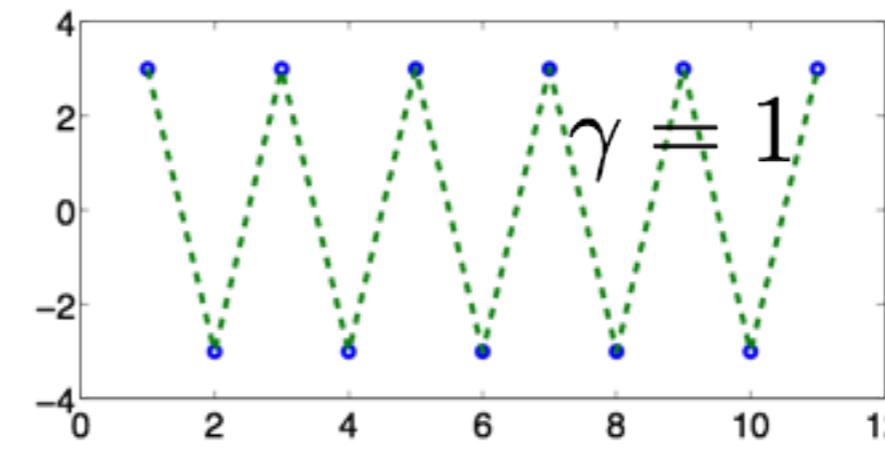
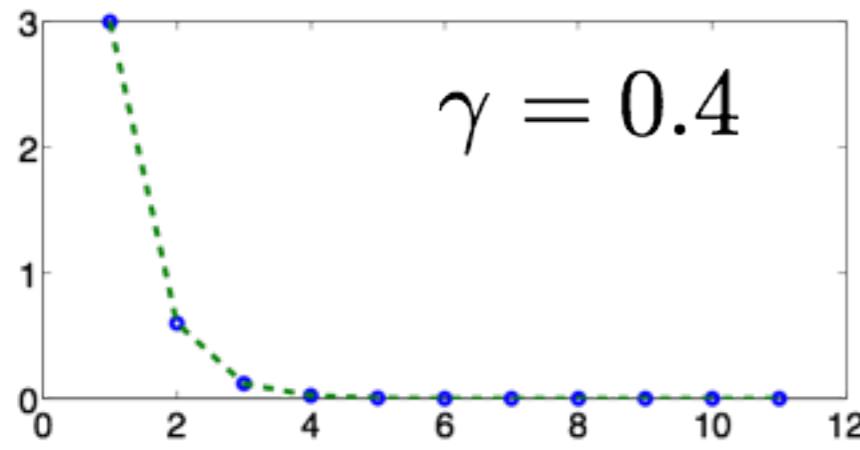
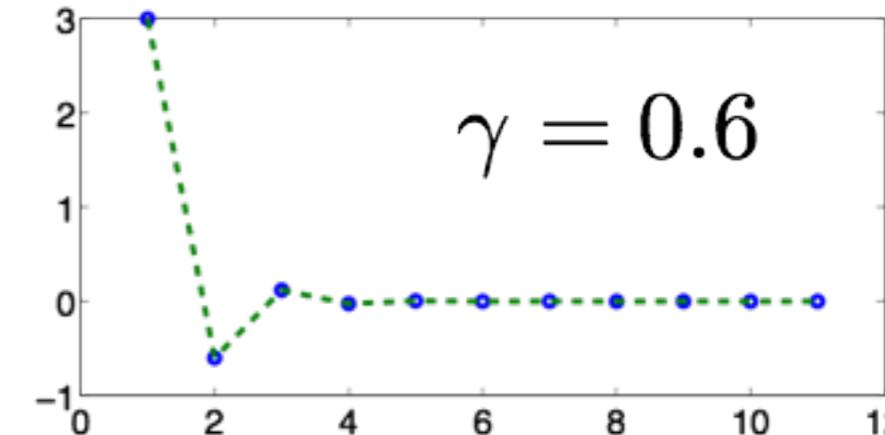
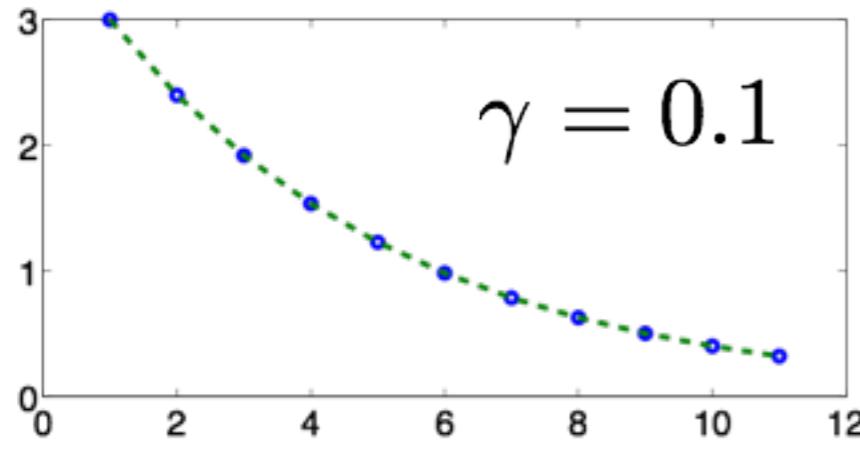


$$-1 < \left[ \frac{dT}{dx} \right]_{x_m} < 1$$

Taylor first order approximation



# Numerical examples



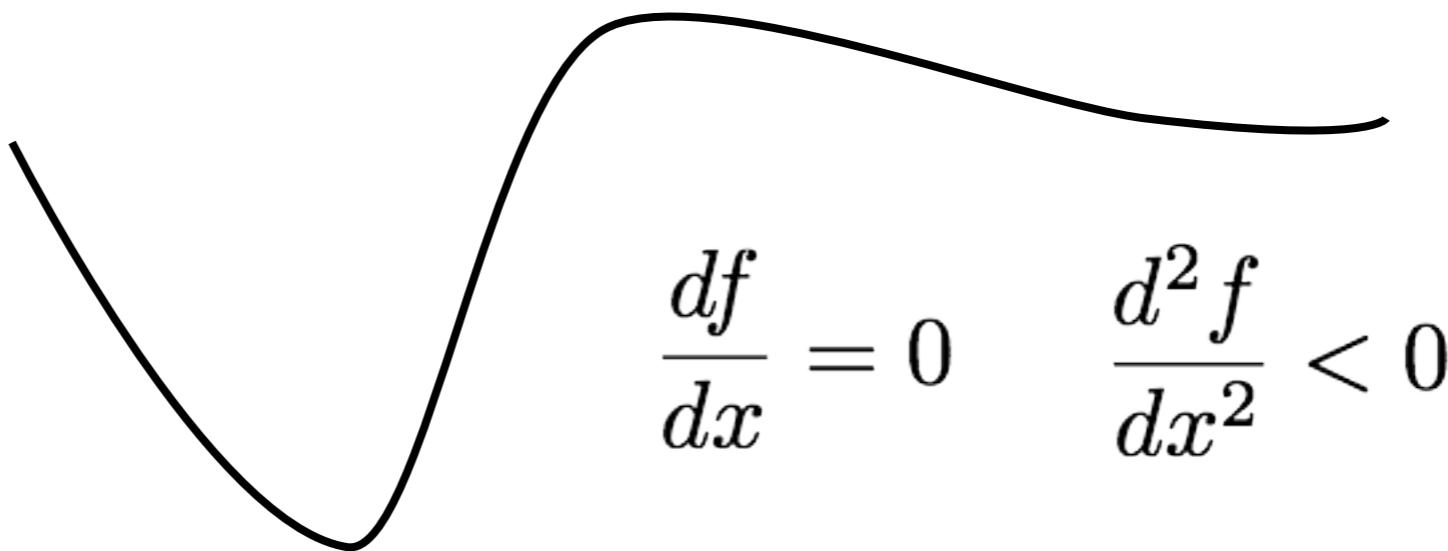
$$C = x^2 + b \quad \Rightarrow \quad \frac{dC}{dx} = 2x$$

$$1 > \frac{d}{dx}(x - \gamma 2x) > -1$$

$$1 > (1 - 2\gamma) > -1 \quad \Rightarrow \quad \gamma \in (0, 1)$$

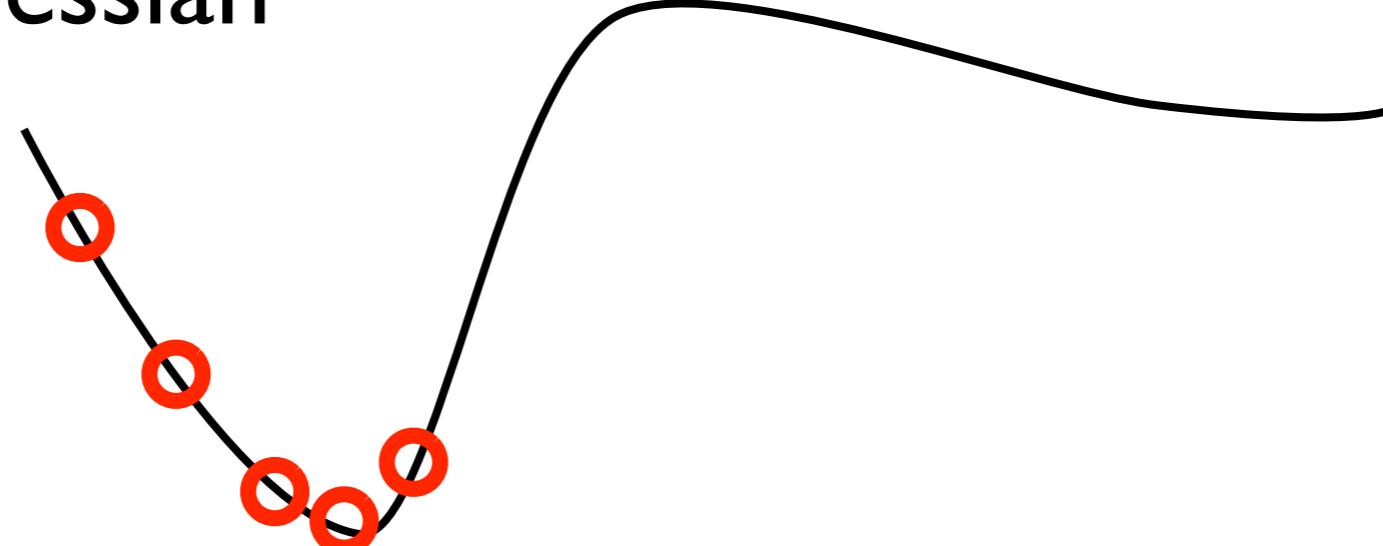
# Stability of gradient descent

Basic stability:



2-nd derivative tells about stability,  
for n-DOF: Hessian

Step size:



Buchli - OLCAR - 2015

# Exploitation vs. Exploration

Choice of learning rate parameter vs. local  
minima, convergence speed

Solution is often adaptive learning rate,  
but...

- freezing too quickly, get stuck in local minimum
- freezing too slowly, slow convergence, wild oscillations in solutions

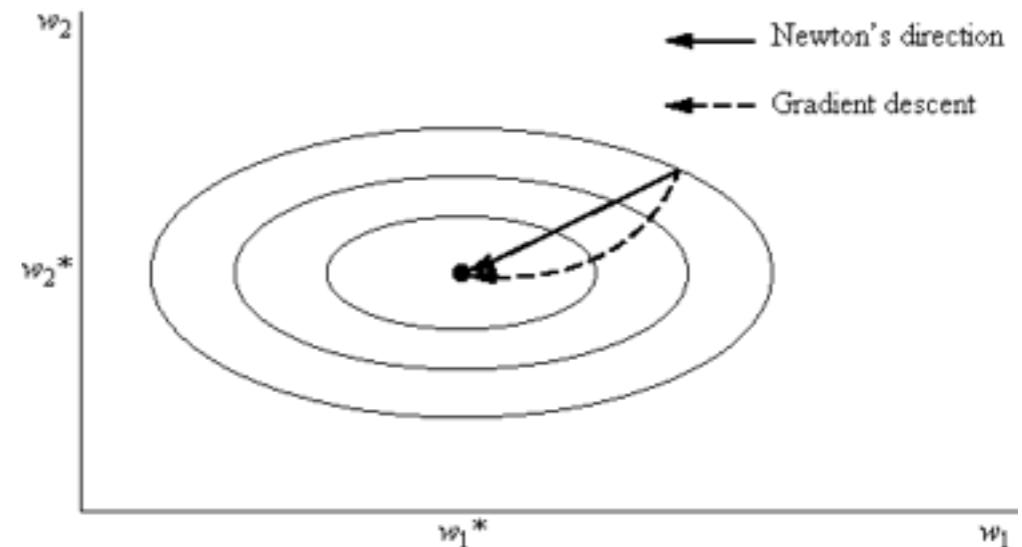
# Newton Method

- Faster convergence
- Less sensitivity to the learning rate

$$x_{m+1} = x_m - \gamma_m (H_c(x_m))^{-1} \nabla C(x_m)$$

$$\nabla C(x) = \frac{\partial C}{\partial x}$$

$$H_c(x) = \frac{\partial^2 C}{\partial x^2}$$



# A real implementation on a robot



Watch on YouTube “Robot Learning to Walk (Toddler)”

<https://www.youtube.com/watch?v=goqWX7bC-ZY>

# Locomotion Skills for Simulated Quadrupeds

Stelian Coros<sup>1,2</sup> Andrey Karpathy<sup>1</sup> Ben Jones<sup>1</sup>  
Lionel Reveret<sup>3</sup> Michiel van de Panne<sup>1</sup>

<sup>1</sup>University of British Columbia

<sup>2</sup>Disney Research Zurich

<sup>3</sup>INRIA, Grenoble University, CNRS



Buchli - OLCAR - 2015



# Flexible Muscle-Based Locomotion for Bipedal Creatures

SIGGRAPH ASIA 2013

**Thomas Geijtenbeek  
Michiel van de Panne  
Frank van der Stappen**



Buchli - OLCAR - 2015



# Problems of FD

- Multiple minima
- Non-smooth cost functions
- performance
- robustness
- step size - exploration vs. exploitation
- noise
- at min. gradient = 0
- when converged?
- Lots of algorithmic parameters
- One update might be computationally intensive